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# *Mathematical Needs of Prospective Students*

COLLEGE OF ENGINEERING · UNIVERSITY OF ILLINOIS

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REVISED EDITION · UNIVERSITY OF ILLINOIS BULLETIN

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# *Mathematical Needs of Prospective Students*

COLLEGE OF ENGINEERING · UNIVERSITY OF ILLINOIS

FOR THE USE OF HIGH SCHOOL MATHEMATICS TEACHERS AND GUIDANCE COUNSELORS

*Revised by a Joint Committee:*

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## *Introduction*

Any professional program of engineering and science education necessarily begins in the secondary schools of the nation. This places teachers of college preparatory subjects in key positions of responsibility. In fact, high quality instruction in English, science, and mathematics is one of the most important contributions the high school can make to the education of future engineers. This bulletin has therefore been prepared to assist with the mathematical preparation of such students.

A significant trend in contemporary American engineering education is the reshaping of curricula so that entering students are required to begin the college study of mathematics at a higher level than in the past and to continue their work beyond the calculus. These requirements, in turn, have demanded a smoother transition between the high school and university instructional programs in mathematics, and have allowed the development of experimental plans in secondary mathematics that are at once broader in coverage and less compartmented than the traditional course sequences. These college curricular changes have also made desirable a digest of at least the minimum content expected of high school mathematics programs so that teachers can have adequate specific information as they prepare their students for professional education in engineering.

The primary purpose of this bulletin is to satisfy this last need by listing topics in algebra, geometry, and trigonometry which the entering engineering student at the University of Illinois is expected to understand and be able to apply. While the course organization and

content of engineering curricula throughout the nation are far from being identical, there is sufficient similarity from institution to institution to support the hope that the list of mathematical competencies given in this bulletin will be helpful to high school counselors and teachers of mathematics throughout the nation, regardless of the engineering college their students plan to attend.

Because of their relevance to the major role of this publication, the subjects of "Programs for Gifted Students" and "Experimental Curricula" are treated, although very briefly. Sample questions from the placement examinations given entering engineering students at the University of Illinois are provided in the appendix to illustrate the general level and character of knowledge expected.

## *Changing Curricula in Colleges of Engineering*

In the last twenty years, applied mathematics has assumed a leading role in engineering research and design. This analytical trend has been accelerated by the development of electronic computers. Automatic computers afford no escape from mathematics, however; on the contrary, they stimulate many research investigations into the theories of approximation and iterative processes. Also, new developments in mathematics filter slowly into technology, thus adding to the total stock of mathematical equipment that practicing engineers must possess. Consequently, the demand for engineering graduates with extensive mathematical training is greater than ever before.

Even though engineering curricula are changing, they retain a hard core of basic courses such as chemistry, general physics, mechanics, thermodynamics, mechanics of materials, structural analysis, fluid mechanics, electric circuits, and electromagnetic fields. These and allied courses determine the immediate mathematical prerequisites of engineering students. To succeed in his engineering assignments, an entering freshman must be familiar at least with the techniques of algebra and trigonometry. Even more important than manipulations, however, are the relations of mathematical ideas to physical situations; many students will attest that they find greater difficulty in setting up equations than in solving them.

## *Historical Antecedents of the Present Bulletin*

In 1951 the College of Engineering of the University of Illinois published the original statement of the mathematical needs of prospective

students.<sup>1</sup> This statement was the result of a thorough study by a University committee consisting of faculty members from the College of Engineering, the Department of Mathematics in the College of Liberal Arts and Sciences, and the College of Education. The committee examined articles in journals and magazines of scientific and engineering societies, as well as books, pamphlets, and research studies by scientists, mathematicians, secondary school teachers, and engineers. These were reviewed to find out what the authors considered necessary mathematical knowledge for successful study of the various engineering curricula.

The group then conducted a series of interviews with students enrolled in the College of Engineering, seeking to determine the usefulness of selected mathematics topics in freshman and sophomore courses. It surveyed all faculty members of the College of Engineering who were teaching freshman or sophomore courses, and a selected staff group in the Department of Mathematics. The purpose was to secure judgments concerning the relative importance for engineering students of various topics in the high school mathematics curriculum.

Using these three sets of data — the findings of other investigators and the two studies conducted at the University of Illinois — the committee selected topics and classified them in two categories: (1) those which were considered indispensable and (2) those which were considered desirable but supplementary. Mathematics teachers of nine representative Illinois high schools were then asked to consider the topics and make suggestions. They were also specifically asked whether in their opinion the set of topics labeled "indispensable" could be taught successfully in a four-year high school mathematics program.

Using the reactions obtained from the teachers, the set of topics was revised, and after being approved by the Policy and Development Committee of the College of Engineering, was printed and distributed.

## *Preparing the Present Revision*

It has been pointed out above that changes are taking place in the curricula of engineering colleges. These changes have implications for the mathematical knowledge of students. By 1958 it had become apparent that the statement of minimum mathematical needs appearing in the 1951 bulletin was in need of revision. Accordingly, a com-

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<sup>1</sup> University of Illinois Bulletin, Volume 49, Number 18: October, 1951.

mittee directed its attention to making appropriate changes in this statement. In revising, the committee continually asked itself: What mathematical knowledge is necessary for successful study in the various engineering curricula?

Answering this question resulted in three kinds of changes. One was the addition of certain topics. For example, instructors in mathematics and in engineering repeatedly remarked on the inability of some students to do simple arithmetical computations mentally. Also, some students seem not to know efficient ways of performing operations even when paper and pencil are used. As an example, some students will use long instead of short division to calculate  $6354 \div 6$ . Such inefficient habits result in a student's not being able to keep up with an instructor's explanations in class. They also waste the student's time when he does his assignments. The committee, therefore, included a statement of these desired skills. Other additions occurred in the list of supplementary topics. These forecast the direction in which college mathematics probably will move.

A second kind of change was the shift of topics no longer considered indispensable to the supplementary list. For example, since any triangle can be solved by either the law of sines or the law of cosines, the law of tangents is used infrequently. The latter topic was therefore shifted from the list of indispensable topics to the list of supplementary topics.

A third kind of change was made by altering the statement of certain topics or their placement in the list to indicate more clearly the nature of the knowledge signified. The topic "Computation by Means of Logarithms" was changed to "Principles of Computation with Logarithms" to indicate that the most important aim is an understanding of how the properties of logarithms facilitate computation rather than prolonged drill in using logarithms for this purpose. It was with this approach in mind that the topic "Solution of Triangles by Logarithms" was deleted. The committee felt that the topic "Solution of Oblique Triangles" would better describe the knowledge desired. As an example of a change in the position of a topic in the list, the topic "Interpolation" was taken from the context of logarithms and placed in the context of ratio and proportion. This was done to indicate that interpolation is to be considered broadly, not narrowly restricted to computing logarithms and antilogarithms.

# *Minimum Mathematical Needs*

This section lists topics in secondary mathematics, an understanding of which is considered to be indispensable for prospective students in the College of Engineering.

It is expected that students who have an understanding of the following topics will be able to begin their mathematics training in college with analytic geometry. These students normally will complete any one of the engineering curricula in four years.

The topics marked with an asterisk are those ordinarily studied in advanced (college) algebra and trigonometry. Students who have an understanding of all topics except those so marked will begin with college algebra and trigonometry as their first mathematics courses in college. These students will take more than the minimum time to complete any of the engineering curricula.

## **INDISPENSABLE TOPICS**

### *Basic Concepts*

1. Skill in performing simple arithmetic and algebraic operations mentally.
2. Skill in performing written operations by the most efficient methods.
3. Fundamental operations with whole numbers, common fractions, decimals, and mixed numbers.
4. Percentage, including per cent of increase and decrease.
5. Ratio and proportion.
6. Interpolation.
7. Measurement, common units of measure, precision of a measurement, accuracy of a measurement, significant digits, and rounding.
8. Conversion of units in a measurement of a physical magnitude.
9. Solution of problems involving measurements, e.g., addition of lengths expressed in feet and inches, calculation of areas and volumes, addition or subtraction of angles.
10. Scale drawing.
11. Constant, variable, parameter.
12. Preparation and interpretation of statistical graphs; viz., bar, circle, and line.

*Algebra*

13. Signs of aggregation; viz., parentheses, brackets, braces, *et al.*, and their use.

14. Rational numbers; i.e., the integers and the fractions.
15. Fundamental operations with rational numbers.
16. Fundamental operations with algebraic fractions.
17. Fundamental operations with polynomials.
18. Common special products; viz.,  $a(b + c)$ ,  $(a + b)(a - b)$ ,  $(a \pm b)^2$ , and  $(a + b)(c + d)$ , emphasizing the distributive law.
19. Factoring; viz.,  $ab + ac$ ,  $a^2 \pm 2ab + b^2$ ,  $a^2 - b^2$ ,  $ax^2 + bx + c$  based on the distributive law.
20. Laws of exponents, including negative and fractional exponents.
21. Solution of linear equations having numerical and/or literal coefficients.
22. Solution of a system of linear equations.
23. Determinants, their evaluation by minors, and their use in solving systems of linear equations.
24. Variation, direct and inverse.
25. Function and functional notation. Representation of a function by a table of corresponding values, by a graph, and, where possible, by an equation or verbal statement.
26. Properties of a linear function; viz., rate of change, graph, slope, and y-intercept of the graph.
27. The quadratic equation: derivation of the quadratic formula; solution by formula and, where appropriate, by factoring.
28. Irrational numbers and fundamental operations with these numbers.
29. Real numbers and fundamental operations with these numbers.
30. Complex numbers and fundamental operations with these numbers.
31. Quadratic polynomials in one variable — standard form, graph, location of maximum or minimum by completing the square; nature of roots, and expressions for the sum and product of the roots of a quadratic equation.
32. Common quadratic equations in two variables.
33. Solution of a system of two quadratic equations.
34. Solution of verbal problems by algebraic methods.
35. Solution of equations in which the unknown occurs under a radical sign.
36. Binomial theorem with positive integral exponents.
37. Scientific notation or standard-form numbers — e.g.,  $2.54 \times 10^3$ ,  $1.2 \times 10^{-4}$ .
38. Principles of computation with logarithms.
- \*39. Change of the base of logarithms.
- \*40. Solution of exponential and logarithmic equations.
- \*41. Factor theorem.

- \*42. Finding the rational roots of higher degree equations of the form  $f(x) = 0$  where  $f(x)$  is a polynomial in  $x$ .
- \*43. Sketching of the graphs of higher degree polynomials.
- \*44. Approximating the irrational roots of higher degree equations, preferably by the method of interpolation.
- \*45. Arithmetic progressions.
- \*46. Geometric progressions, both finite and infinite.
- 47. Properties of the relation of equality.
- 48. Properties of the relation of inequality.

### *Geometry*

- 49. Use of the protractor.
- 50. Use of the compass and straight edge in making simple geometric constructions.
- 51. Plane angle.
- 52. Dihedral angle.
- 53. Polygons: triangle, rectangle, parallelogram, trapezoid, hexagon, octagon.
- 54. Circles, including the construction of circles tangent to lines and to each other.
- 55. Angle inscribed in a semicircle.
- 56. Mensuration of plane figures.
- 57. Congruence.
- 58. Similarity.
- 59. Symmetry.
- 60. Locus.
- 61. Parallelism and perpendicularity of lines.
- 62. Pythagorean theorem.
- 63. Projection.
- 64. Pictorial representation of three-dimensional figures on a plane.
- 65. Parallelism and perpendicularity of a line and a plane.
- 66. Parallelism and perpendicularity of planes.
- 67. Mensuration of solid figures; viz., cubes, prisms, pyramids, cylinders, cones, and spheres.
- 68. Definition, postulate, theorem.
- 69. Deductive proof.
- 70. Inductive reasoning: its use in science and mathematics, and the difference between inductive reasoning and proof.

### *Trigonometry*

- 71. Trigonometric functions of an acute angle.
- 72. Values of the functions of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ .

73. Solution of right triangles.  
 74. Trigonometric functions of complementary angles:  $\sin (90^\circ - A) = \cos A$ , *et al.*  
 75. Solution of verbal problems involving right triangles.  
 \*76. Definitions of trigonometric functions of any angle.  
 \*77. Values of functions for special angles, including quadrantal angles.  
 \*78. Numerical value of functions of any angle from trigonometric tables, natural and logarithmic.  
 \*79. Fundamental trigonometric identities.  
 \*80. Addition identities:  $\sin (A + B)$ , *et al.*  
 \*81. Law of sines.  
 \*82. Law of cosines.  
 \*83. Area formulas:  

$$K = \frac{1}{2} bc \sin A; K = \sqrt{s(s - a)(s - b)(s - c)}$$

\*84. Solution of oblique triangles.  
 \*85. Radian measure of angles.  
 \*86. Graphs of sine and cosine functions.  
 \*87. Inverse trigonometric functions.  
 \*88. Solution of trigonometric equations.  
 \*89. Double angle identities.  
 \*90. Half angle identities.  
 \*91. Proofs of identities.  
 \*92. Vector, component, and resultant.  
 \*93. Graphical addition and subtraction of vectors.  
 \*94. Addition and subtraction of vectors by components.

#### **SUPPLEMENTARY TOPICS**

Some topics are not sufficiently fundamental to be classified as indispensable. It is recommended, however, that the subjects listed below be studied if there is time available for the whole group or for individual students whose rate of learning warrants supplementary work.

1. Extraction of square roots.
2. Binomial theorem with fractional and negative exponents.
3. Permutations.
4. Combinations.
5. Probability.
6. Multiplication and division of complex numbers in polar form.
7. De Moivre's theorem.
8. Exponential form of a complex number.
9. Ordered pair form of a complex number.

10. Line values of trigonometric functions.
11. Tangents of half-angle formulas.
12. Law of tangents.
13. Inverse, converse, and contra-positive of a statement.
14. Polyhedral angles.
15. Slide rule theory and operation (will be used in college).
16. Binary numeration.
17. Set, element of a set, designation of a set by description and listing; set-builder.
18. Subset, proper subset.
19. Empty set, complement of a set.
20. Operations on sets: union and intersection.
21. Ordered pair of numbers, set of ordered pairs of numbers, Cartesian set.
22. Open sentences, statements.
23. Relation as a set of ordered pairs of numbers.
24. Further development of the function concept, i.e., as a set of ordered pairs of numbers in which each element of the domain is paired with one and only one element of the range; inverse of a function.
25. Descriptive statistics: measures of central tendency and simple measures of dispersion.
26. Properties of a number field, examples of fields.
27. Circular functions of real numbers, certain inverse circular functions; viz., arc sin, arc cos, arc tan.
28. Derivative of a polynomial, inverse of a derivative.

## *How the Minimum List Can Be Used*

The teacher can apply the list to direct the study of those who contemplate entering a college of engineering. Since many related fields of scientific study (such as physics, chemistry, or mathematics) require about the same high school preparation, it is expected that advanced high school mathematics classes will contain many students for whom the list can serve as a guide.

Although not all students in secondary school will have decided upon their careers, enthusiasm for the study of mathematics can be recognized as a broad pattern of interest, one of several which apparently remain stable over considerable periods of time. Various tests

can be used to identify these patterns; one which finds favor with high schools is the Kuder Preference Record. If at all possible, this test or a similar one should be given to the whole group of students. Then, those who show a strong interest in science and who are taking advanced mathematics can be supplied with the list of topics developed by the committee. Indeed, it would be well to provide each such student with a reproduced copy of the list.

One thing which must be stressed is that this list of mathematical needs is a *minimum* list. *It would be unfortunate if the list became the maximum content of a four-year program in mathematics. In order to provide the students with a thorough and broad understanding of mathematics, it is necessary to go beyond the foregoing list of minimum essentials. The teacher should encourage each student to progress as far as his capabilities will allow.* (See page 16, "Programs for Gifted Students.")

In schools which have centralized guidance programs, the list may be used by other faculty members in addition to the mathematics teacher. There may be one person — the principal, the assistant principal, or the counselor — who advises students on educational and career choices. Such a person should find the list helpful in advising students who are interested in preparing to enter a college of engineering or who plan to train for other careers requiring comparable mathematical skills.

For a student who is considering engineering as a possible profession, the list can be presented to illustrate one kind of knowledge, understanding, and skill he must have to succeed in an engineering curriculum. The student, his parents, and the adviser should then consider seriously and as objectively as possible the information available to them about the student's potentialities in the light of what is required to succeed in engineering. The resulting decision, whether to choose engineering or not, ought to be a wiser one than if the decision were made without considering the demands of an engineering career as compared with the student's real interests and abilities.

A teacher or adviser will also find helpful the *Guidance Pamphlet in Mathematics for High School Students* published by the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.; *Why Study Mathematics?*, published by the Canadian Mathematical Congress, Engineering Building, McGill University, Montreal, Canada; *Three Whys and Math at General Electric*, published by the General Electric Company, 1 River Road, Schenectady 5, New York, and *Career Opportunities in Mathematics*, pub-

lished by the Mathematical Association of America, University of Buffalo; Buffalo 14, New York. These publications set forth the place of mathematics in many occupations, including engineering. When made available to a student, they will help him decide which occupation he wants to select.

## *Mathematics Entrance Requirements for the College of Engineering, University of Illinois*

The mathematics entrance requirements of the College of Engineering are two units of algebra, one unit of plane geometry, and one-half unit of trigonometry. It is recommended that students also study solid geometry in high school. Some secondary schools offer instruction in all of these subjects, either in the form of the traditional courses or in four-year mathematics programs which have eliminated the compartmentation of subject matter. An example of the latter is the program being developed by the University of Illinois Committee on School Mathematics.

Since September 1, 1953, analytic geometry has been the first course in mathematics for which credit is granted toward graduation in all four-year engineering curricula.

In order to enroll in analytic geometry in their first semester at Illinois, students must:

1. Have college credit in college algebra and trigonometry, or
2. Pass a placement test in mathematics.

The placement examination is given to all entering students presenting three and one-half or four units of high school mathematics. This test assesses the student's understanding of those topics contained in the list of minimum mathematical needs of prospective students in the College of Engineering (items 1-94, pages 9-12). Topics in the list that are marked with an asterisk, as well as those not so marked, are covered in the test. In addition, special provision is made for appropriate placement of students who have had substantially more mathematics than is included in the usual high school program.

Students may be admitted to the College of Engineering with a minimum of one unit each in algebra and plane geometry, but they are required to make up the deficiencies during their first year in college.

In general, students who are deficient in high school mathematics may prepare themselves to enroll in analytic geometry in one of the following ways:

1. By passing college algebra and trigonometry in their first semester at Illinois. These students will require a minimum of a summer session in addition to the regular eight-semester program to complete requirements for an engineering degree. In most cases the students prefer to attend a summer session following the freshman year to take differential calculus and the first course in physics.
2. By attending a summer session at an accredited institution to take college algebra and trigonometry before enrolling for the first full semester of University work.
3. By enrolling for appropriate mathematics courses offered by the Extension Division of the University of Illinois<sup>1</sup> or other accredited institutions.
4. By self-study in the subjects they have not taken in high school to prepare for (a) the mathematics placement test<sup>2</sup> mentioned previously or (b) proficiency examinations in the individual mathematics courses.

It can be seen that — assuming proper guidance, aptitude, and ambition — the mathematics requirements do not present undue difficulties for any high school student who wishes to pursue any engineering curriculum at the University of Illinois. At the same time, these requirements permit and indeed encourage latitude on the part of high school teachers in planning mathematics programs.

## *Programs for Gifted Students*

The use of ability grouping for classes in a substantial number of high schools has made possible the development of programs for gifted students. Such programs assist these students in obtaining advanced preparation for college. It is well established that gifted students can cover more material than is usually presented in four years of high school mathematics. Indeed, to challenge and interest such students, some form of advanced work is highly desirable. Suitable provision

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<sup>1</sup>See the University of Illinois bulletin *Correspondence Courses* for a description of course offerings, regulations, and procedures for admission.

<sup>2</sup>See appendix for sample examination questions.

for these students is welcomed by the University, which itself offers an honors program in mathematics for superior students, including freshmen.

The two methods outlined below are those commonly used in programs for gifted students.

#### *Supplementation*

Here a substantial amount of supplementary material is added to the normal curriculum each year. In the last two years, this can include a fairly extended treatment of probability and statistics, an introduction to the simple aspects of modern mathematics such as logic, set theory, and modern algebra, or a start with the concepts of analytic geometry and calculus.

#### *Acceleration*

Here the normal curriculum is accelerated so that the gifted student has mastered the list of minimum mathematics needs by the end of the junior year. Part or all of the senior year is then devoted to a college-level course in analytic geometry or in analytic geometry and calculus. Obviously the goal here is to have the student achieve advanced placement in mathematics upon entering the University. Some high schools which use this second approach also offer an elective senior course in modern mathematics, and others have mathematics clubs in which supplementary material is discussed. Thus the benefits of both philosophies can be gained from one program.

In general, the College of Engineering favors the second of the two methods, but endorses both and their combinations. Naturally, the success or advisability of such programs for gifted students depends upon having both a strong teaching staff and a significant number of students who can profit from advanced work. Experience has shown that some 10 to 15 per cent of high school freshmen can carry stronger programs successfully, though this percentage may vary considerably from school to school.

The College of Engineering wishes to commend high schools which offer programs for gifted students and to encourage other well-equipped schools to explore the possibility of initiating such programs. One very successful arrangement which takes advantage of acceleration is described below.

#### **ADVANCED PLACEMENT PROGRAM ADMINISTERED BY THE COLLEGE ENTRANCE EXAMINATION BOARD**

The Fund for the Advancement of Education several years ago sponsored an effort to strengthen education for gifted students by

establishing college-level courses in various disciplines for selected high school seniors. Since 1955 these courses have been administered by the College Entrance Examination Board. In mathematics, the Advanced Placement Program specifies a year of analytical geometry and calculus, followed by a three-hour examination prepared under the auspices of the Board. Results from this examination are used by the University of Illinois in determining college placement and credit for such students.

Further information about the Program can be obtained from the Director, Advanced Placement Program, College Entrance Examination Board, 425 West 117th Street, New York 27, New York.

## *Experimental Curricula*

There are under development various new programs of high school mathematics, the primary purpose of which is to close the gap between secondary-school mathematics and the modern mathematics that is currently proving useful in an ever-increasing number of applications. Among these programs are:

1. The detailed list of recommendations by the Commission on Mathematics of the College Entrance Examination Board, prepared under the direction of Dean Albert E. Meder.
2. The experimental classroom materials being developed by the University of Illinois Committee on School Mathematics (UICSM) at the University High School under the direction of Professor Max Beberman.
3. The School Mathematics Study Group under the direction of Professor E. G. Begle at Yale University.
4. The Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, under the direction of Mr. Frank Allen.

In contrast to much conventional teaching, all these programs propose the inclusion of new material in the high school offerings as well as new approaches to familiar materials. They also place greater emphasis on the presentation and comprehension of *ideas* than is usually the case, but not to the extent of excluding the mastery of essential techniques.

These aims are universally recognized as significant, indeed urgent; hence there is no intention of appearing to diminish the importance

of these experiments in any way. It should be emphasized, however, that such programs *are still experimental*. Therefore, although it is expected that these programs will ultimately contribute significantly to the effectiveness, the usefulness, and the richness of the mathematics taught in our high schools, there is no need for school boards or teachers to feel compelled to make hasty or ill-considered shifts to a new type of mathematics curriculum. The fact is that any teacher who presents the customary courses in a careful and inspiring manner, emphasizing ideas and understanding as well as technique, is properly preparing students for standard college-level work and may continue to take pride in his efforts. Indeed, his contribution will be greater than that of the teacher who is forced to adopt approaches and materials for which he has not been adequately trained. Only as the feasibility and the usefulness of these new approaches and new materials are demonstrated need they be incorporated into the curriculum. Moreover, this should be done only after due preparation on the part of the teachers involved.

## *Books Suggested for the High School Library as Sources of Supplementary Materials*

Abbott, E. A. *Flatland*. New York: Dover, 1952.

Allendoerfer, C. B., and Oakley, C. O. *Principles of Mathematics*. New York: McGraw-Hill Book Co., 1955.

Andree, R. V. *Selections from Modern Abstract Algebra*. New York: Henry Holt and Co., 1958.

Archibald, R. C. *Outline of the History of Mathematics*. Buffalo: Mathematical Association of America, 1940.

Ball, W. W. R., and Coxeter, H. S. M. *Mathematical Recreations and Essays*. New York: Macmillan, 1939.

Begle, E. G. *Introductory Calculus with Analytic Geometry*. New York: Henry Holt, 1954.

Bell, E. T. *Mathematics, Queen and Servant of Science*. New York: McGraw-Hill Book Co., 1951.

Bell, E. T. *Men of Mathematics*. New York: McGraw-Hill Book Co., 1937.

Bell, E. T. *The Development of Mathematics*. New York: McGraw-Hill Book Co., 1940.

Birkhoff, G. D., and Beatley, R. *Basic Geometry*. New York: Chelsea Publishing Company, 1958.

Birkhoff, G., and MacLane, S. *A Survey of Modern Algebra*. New York: Macmillan, 1953.

Breuer, J. (translated by H. F. Fehr). *Introduction to the Theory of Sets*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1958.

Carnahan, W. *Mathematics Clubs in High School*. Washington, D.C.: The National Council of Teachers of Mathematics.

Chernoff, H., and Moses, L. *Statistics: Decision Making Under Uncertainty*. Palo Alto, California: Stanford University Bookstore, 1958.

Christian, R. R. *Introduction to Logic and Sets*. Boston: Ginn and Company, 1958.

Commission on Mathematics. *Introductory Probability and Statistical Inference*. New York: College Entrance Examination Board, 1957.

Cooley, H. R., Gans, D. G., Kline, M., and Wahlert, H. E. *Introduction to Mathematics*. Boston: Houghton Mifflin Co., 1949.

Courant, Richard. *Differential and Integral Calculus*. Volumes I and II. New York: Interscience, 1950.

Courant, R., and Robbins, H. *What Is Mathematics?* New York: Oxford University Press, 1948.

Dubisch, R. *The Nature of Number*. New York: Ronald Press, 1952.

Dubisch, R. *Trigonometry*. New York: Ronald Press, 1955.

Eves, H., and Newsom, C. V. *An Introduction to the Foundations and Fundamental Concepts of Mathematics*. New York: Rinehart and Co., 1958.

Hartley, M. C. *Patterns of Polyhedrons*. Ann Arbor, Michigan: Edwards Brothers, 1948.

Hilbert, D., and Cohn-Vossen, S. *Geometry and the Imagination*. New York: Chelsea Publishing Co., 1956.

Huff, D., and Geis, I. *How to Lie with Statistics*. New York: W. W. Norton Co., 1954.

Johnson, D. *Paper Folding for the Mathematics Class*. Washington, D.C.: The National Council of Teachers of Mathematics, 1957.

Johnson, R. E., and Kiokemeister, F. L. *Calculus with Analytic Geometry*. Boston: Allyn and Bacon, Inc., 1957.

Jones, B. W. *Elementary Concepts of Mathematics*. New York: Macmillan, 1947.

Jones, S. I. *Mathematical Clubs and Recreations*. Nashville, Tennessee: Jones, 1940.

Kasner, E., and Newman, J. *Mathematics and the Imagination*. New York: Simon and Schuster, 1940.

Kells, L. M., Kern, W. F., and Bland, J. R. *Plane and Spherical Trigonometry*. New York: McGraw-Hill Book Co., 1951.

Kemeny, J. G., Snell, J. L., and Thompson, G. L. *Introduction to Finite Mathematics*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1957.

Kershner, R. B., and Wilcox, L. R. *The Anatomy of Mathematics*. New York: Ronald Press, 1950.

Kline, Morris. *Mathematics in Western Culture*. New York: Oxford University Press, 1953.

Kraitchik, M. *Mathematical Recreations*. New York: Dover Publications, 1953.

Lieber, L. R., and Lieber, H. G. *Mits, Wits and Logic*. New York: Norton, 1947.

Lieber, L. R., and Lieber, H. G. *Take a Number: Mathematics for the Two Billion*. Lancaster, Pa.: Jaques Cattell Press, 1946.

Lieber, L. R., and Lieber, H. G. *The Education of T. C. Mits*. New York: Norton, 1944.

May, K. O. *Fundamental Mathematics*. Reading, Mass.: Addison-Wesley Publishing Co., 1958.

Menger, Karl. *Basic Concepts of Mathematics*. Chicago: Illinois Institute of Technology Bookstore, 1957.

Middlemiss, R. R. *Analytic Geometry*. New York: McGraw-Hill Book Co., 1955.

National Council of Teachers of Mathematics. *Insights into Modern Mathematics*. 23rd Yearbook. Washington, D.C.: The National Council of Teachers of Mathematics, 1957.

National Council of Teachers of Mathematics. *The Mathematics Student Journal*. Washington, D.C.: The National Council of Teachers of Mathematics. (This is a periodical published for students.)

Newman, J. R. *The World of Mathematics*. Four volumes. New York: Simon and Schuster, 1956.

Northrop, E. *Riddles in Mathematics*. New York: Van Nostrand, 1944.

Ore, O. *Number Theory and Its History*. New York: McGraw-Hill Book Co., 1948.

Polya, G. *How to Solve It*. Garden City, New York: Doubleday Anchor Books (paperback), 1957.

Polya, G. *Plausible Reasoning*. Volumes I and II. Princeton, N.J.: Princeton University Press, 1954.

Rademacher, H., and Toeplitz, O. *The Enjoyment of Mathematics*. Princeton, N.J.: Princeton University Press, 1957.

Raiffa, H., and Luce, D. *Theory of Games and Decision Making*. New York: John Wiley and Sons, 1957.

Richardson, M. *Fundamentals of Mathematics*. New York: Macmillan, 1958.

Sanford, V. A. *A Short History of Mathematics*. Boston: Houghton Mifflin, 1930.

Sherwood, G. E. F., and Taylor, A. E. *Calculus*. Englewood Cliffs, N.J.: Prentice-Hall, 1954.

Shuster, C. N., and Bedford, F. L. *Field Work in Mathematics*. East Palestine, Ohio: Yoder Instrument Company.

Sisam, C. H., and Atchison, W. F. *Analytic Geometry*. New York: Henry Holt, 1955.

Smith, D. E. *History of Mathematics*. Volumes I and II. New York: Dover Publications, 1951.

Smith, D. E. *Source Book of Mathematics*. Cambridge, Mass.: Harvard University Press, 1956.

Stabler, E. R. *Introduction to Mathematical Thought*. Cambridge: Addison-Wesley Press, 1953.

Suppes, P. C. *Introduction to Logic*. Princeton, N.J.: Van Nostrand Co., 1957.

Swain, R. L. *Understanding Arithmetic*. New York: Rinehart and Co., 1957.

University of Chicago College Mathematics Staff. *Concepts and Structure of Mathematics*. Chicago: The University of Chicago Press, 1954.

University of Illinois Committee on School Mathematics. *High School Mathematics*. Urbana, Illinois.

Uspensky, J. V. *Theory of Equations*. New York: McGraw-Hill Book Co., 1948.

Weiss, M. *Higher Algebra for the Undergraduate*. New York: John Wiley and Sons, 1949.

Williams, W. T., and Savage, G. H. *The Penguin Problems Book*. New York: Penguin Books, 1940.

Williams, W. T., and Savage, G. H. *The Second Penguin Problems Book*. New York: Penguin Books, 1944.

Wolfe, H. E. *Introduction to Non-Euclidean Geometry*. New York: Dryden Press, 1945.

Yates, R. C. *The Trisection Problem*. Ann Arbor, Michigan: Edwards Brothers Press, 1947.

# Appendix

## SAMPLE PLACEMENT EXAMINATION QUESTIONS

1. Which one of the following is not equivalent to  $\frac{3}{4}$ ?
  - (a)  $\frac{-3}{-4}$
  - (b)  $-\frac{(-3)}{(4)}$
  - (c)  $-\frac{(3)}{(-4)}$
  - (d)  $-\frac{(-3)}{(-4)}$
  - (e)  $-\left(-\frac{3}{4}\right)$
2.  $\frac{x^2 - y^2}{x - y}$  equals
  - (a)  $x - y$
  - (b)  $x + y$
  - (c)  $\frac{1}{x} - \frac{1}{y}$
  - (d)  $\frac{1}{x} + \frac{1}{y}$
  - (e) None of these
3. Which one of the following is always less than 1 when  $x$  is positive?
  - (a)  $\frac{1+x}{x}$
  - (b)  $\frac{x}{1-x}$
  - (c)  $\frac{x}{1+x}$
  - (d)  $x^2$
  - (e)  $\frac{x}{10}$
4. If  $f(x) = x^2 + 6$ , what does  $f(-2)$  represent on the graph of  $f(x)$ ?
  - (a) The point on the  $x$ -axis where  $x = -2$
  - (b) The abscissa of the point on the curve for which  $x = -2$
  - (c) The ordinate of the point on the curve for which  $x = -2$
  - (d) The  $y$ -intercept
  - (e) The  $x$ -intercept

5. A fraction equal to  $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$  with rational denominator may be obtained by

- (a) Multiplying the denominator by  $\sqrt{5} + \sqrt{3}$
- (b) Multiplying the numerator and denominator by  $\sqrt{5} - \sqrt{3}$
- (c) Multiplying the numerator and denominator by  $\sqrt{5} + \sqrt{3}$
- (d) Squaring the numerator and denominator
- (e) Multiplying the denominator by  $\sqrt{5} - \sqrt{3}$

6. The reciprocal of  $\frac{1}{x} + \frac{1}{y}$  is

- (a)  $x + y$
- (b)  $\frac{x + y}{xy}$
- (c)  $\frac{xy}{x + y}$
- (d)  $\frac{1}{x + y}$
- (e) None of these

7.  $\frac{x^{-2} - y^0}{x^{-2} + y^0}$  is equal to

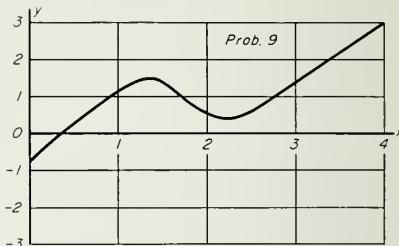
- (a)  $\frac{1 - x^2}{1 + x^2}$
- (b)  $\frac{1 - x^2y}{1 + x^2y}$
- (c)  $-1$
- (d)  $1$
- (e) None of these

8. Another expression for 0.00092 is

- (a)  $92 \times 10^4$
- (b)  $92 \times 10^3$
- (c)  $92 \times 10^{-3}$
- (d)  $92 \times 10^{-4}$
- (e)  $92 \times 10^{-5}$

9. Which one statement of the following about the graph shown below is correct throughout the range  $x=0$  to  $x=4$ ?

- (a) For each value of  $x$  there is only one value of  $y$
- (b) For each value of  $y$  there is only one value of  $x$
- (c)  $y$  is greater than zero for all values of  $x$
- (d)  $y$  increases as  $x$  increases
- (e)  $x$  changes more rapidly than  $y$



10. Which one of the following statements about the function  $y=1-4x^2$  is not true?

- (a) This is a quadratic function
- (b) The function has a minimum value
- (c) The  $y$ -intercept is 1
- (d) The  $x$ -intercepts are  $\frac{1}{2}$  and  $-\frac{1}{2}$
- (e) The graph of the function is symmetrical about the  $y$ -axis

11. If  $N = \frac{2 \cdot \sqrt{3}}{5}$  then  $\log N$  is

- (a)  $\frac{\log 2 \cdot \log \sqrt{3}}{\log 5}$
- (b)  $\frac{\log 2 + \frac{1}{2} \log \sqrt{3}}{\log 5}$
- (c)  $\log 2 + \frac{1}{2} \log 3 - \log 5$
- (d)  $\log 2 + 2 \log 3 - \log 5$
- (e) None of these

12. If  $3^{-x} = \frac{1}{9}$  then  $x$  is

- 2
- 2
- $\frac{1}{2}$
- $-\frac{1}{2}$
- Imaginary

13. The equation  $b^x = n$  is equivalent to

- $\log_b x = n$
- $\log_x b = n$
- $\log_x n = b$
- $\log_b n = x$
- None of these

14. What is the value of  $x$  in the equation  $2^{x+1} = 32^{2-x}$ ?

- 4
- 1.5
- 0.5
- 1.5
- None of these

15. What is the root of  $\frac{x-1}{3} = ax$ ?

- $3ax + 1$
- $\frac{x-1}{3a}$
- $\frac{1}{3a-1}$
- $\frac{1}{1-3a}$
- None of these

16. The expression "s is a function of  $t$ " means

- $s$  is a multiple of  $t$
- $t$  is a multiple of  $s$
- $s$  equals  $t$
- $s$  varies directly as  $t$
- The value of  $s$  depends on the value of  $t$

17.  $r^{2/3} \cdot r^{1/2}$  equals

- $r^{1/3}$
- $r^{3/5}$
- $r^{7/6}$
- $r^{4/3}$
- None of these

18. If  $x+4$  is a factor of  $x^3+4x^2-x+2k$ , what is the value of  $k$ ?

- 4
- 2
- 0
- 4
- None of these

19. Which of the following is not a possible rational root of  $6x^3-5x^2+7x+3=0$ ?

- $\frac{1}{3}$
- $\frac{1}{2}$
- $\frac{3}{2}$
- 3
- 6

20. In the formula  $F = \frac{km_1m_2}{r^2}$ ,  $k$  is a constant. If  $m_1$  and  $m_2$  increase and  $r$  decreases, how will  $F$  change?

- Increase
- Decrease
- Remain the same
- First increase, then decrease
- Impossible to determine

21. The value of  $\frac{x-1}{x}$  when  $x = i = \sqrt{-1}$  is

- 1
- $i$
- $1+i$
- $i-1$
- None of these

22. "Given a straight line  $AB$  and a point  $C$  not on the line; through  $C$ , one and only one line can be drawn parallel to  $AB$ ."

The above statement is an example of

- A definition
- A theorem
- A corollary
- A postulate or axiom
- None of these

23. How many degrees are in each angle of a regular hexagon?

- $60^\circ$
- $105^\circ$
- $120^\circ$
- $135^\circ$
- None of these

24. One angle of a quadrilateral inscribed in a circle is  $50^\circ$ . The opposite angle is

- $140^\circ$
- $130^\circ$
- $120^\circ$
- $40^\circ$
- None of these

25.  $AB$  is the hypotenuse of a right triangle. The locus of the third vertex in a plane is

- The perpendicular bisector of  $AB$
- Two lines parallel to  $AB$
- A circle with  $AB$  as radius
- A circle with  $AB$  as diameter
- None of these

26. A cube has exactly

- 16 edges and 8 faces
- 16 edges and 6 faces
- 12 edges and 8 faces
- 8 edges and 6 faces
- None of these

27. If the radius of one sphere is one-half the radius of another, the ratio of their volumes is

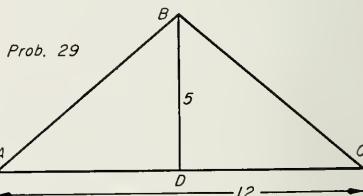
- $\frac{1}{8}$
- $\frac{1}{6}$
- $\frac{1}{4}$
- $\frac{1}{2}$
- None of these

28. What is the locus of all points in space at a distance  $d$  from line  $AB$ ?

- Two lines, one on each side of  $AB$  at a distance  $d$  from  $AB$
- One line parallel to  $AB$  at a distance  $d$  from  $AB$
- The surface of a cylinder whose axis is  $AB$  and whose radius is  $d$
- The perpendicular bisector of  $AB$
- None of these

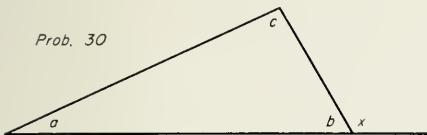
29.  $ABC$  is an isosceles triangle. Its altitude  $BD=5$  and its base  $AC=12$ . What is the approximate length of  $AB$ ?

- 13.0
- 10.9
- 7.8
- 7.5
- 6.9



30. The angle  $x$  equals

- (a)  $a + b$
- (b)  $a + c$
- (c)  $b + c$
- (d)  $a + b - c$
- (e) None of these



31. Which one of the following equations is an identity?

- (a)  $\sin \theta + \cos \theta = 1$
- (b)  $\cos^2 \theta - \sin^2 \theta = 1$
- (c)  $\cos^2 \theta + 1 = \sin^2 \theta$
- (d)  $\sin^2 \theta - \cos^2 \theta = 1$
- (e)  $1 - \cos^2 \theta = \sin^2 \theta$

32. If the two legs of a right triangle are of unequal length, the angle opposite the smaller leg will have a magnitude limited by the range of

- (a)  $0^\circ$  to  $90^\circ$
- (b)  $0^\circ$  to  $60^\circ$
- (c)  $0^\circ$  to  $45^\circ$
- (d)  $45^\circ$  to  $90^\circ$
- (e)  $45^\circ$  to  $135^\circ$

33. The sine of  $45^\circ$  is

- (a)  $\frac{1}{2}$
- (b)  $\frac{1}{2} \sqrt{2}$
- (c)  $\frac{1}{2} \sqrt{3}$
- (d) 1
- (e)  $\sqrt{2}$

34. The tangent of  $60^\circ$  is best approximated by

- (a) 0.500
- (b) 0.577
- (c) 0.755
- (d) 0.866
- (e) 1.732

35.  $\cos(A - B)$  is equal to

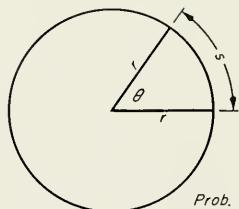
- (a)  $\cos A \cos B + \sin A \sin B$
- (b)  $\cos A \cos B - \sin A \sin B$
- (c)  $\sin A \cos B + \cos A \sin B$
- (d)  $\sin A \cos B - \cos A \sin B$
- (e) None of these

36. When solving an oblique triangle by means of the law of cosines, which one of the following is correct?

- (a)  $a = b + c - 2bc \cos A$
- (b)  $a^2 = b^2 + c^2 + 2bc \cos A$
- (c)  $a^2 + b^2 + c^2 = 2bc \cos A$
- (d)  $a^2 = b^2 + c^2 - 2bc \cos A$
- (e)  $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$

37. Which one of the following is correct?

- (a)  $s = \frac{\theta}{r}$
- (b)  $s = \frac{r}{\theta}$
- (c)  $\theta = \frac{s}{r}$
- (d)  $\theta = \frac{r}{s}$
- (e)  $s = 2\pi r$



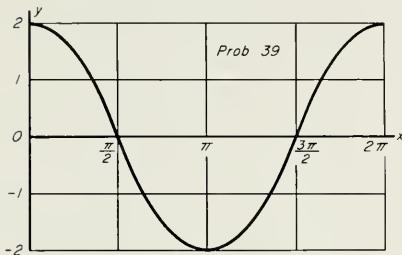
Prob. 37

38. Which trigonometric functions are both positive in the fourth quadrant?

- (a) sine and cosine
- (b) sine and cosecant
- (c) cosine and tangent
- (d) tangent and cotangent
- (e) cosine and secant

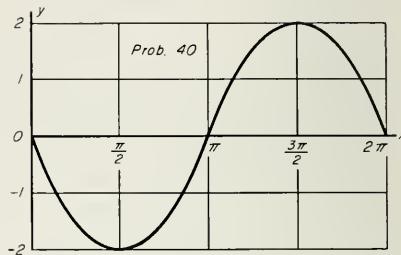
39. The figure shows the graph of which one of the following equations?

- (a)  $y = -2 \sin x$
- (b)  $y = 2 \sin x$
- (c)  $y = 2 \cos x$
- (d)  $y = -2 \cos x$
- (e) None of these



40. The figure shows the graph of which one of the following equations?

- (a)  $y = -2 \sin x$
- (b)  $y = 2 \sin x$
- (c)  $y = 2 \cos x$
- (d)  $y = -2 \cos x$
- (e) None of these













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